public revenue is disbursed every year in the form of life annuities; and that about £600,000 on the average is annually invested in that mode with the Government. The amount of these investments fluctuates considerably; and curiously enough the fluctuations are a very fair index of the state of the money market at the time. Thus, in the 15 years under review, it will be observed that the greatest amount of annuities was granted in 1858, the year immediately succeeding the suspension of the Bank Charter Act in November 1857; and the least in 1864, a period of reckless speculation, the reaction from which is clearly discernible in the largely increased transactions of the following year.

German Life Assurance Institute.

IN accordance with the announcement we made in our last Number, we now extract from the proceedings of the German Life Assurance Institute the two following papers.—ED. *J. I. A.*

I.—*4th Feb.* 1868. DR. ZILLMER *on the Arithmometer, or calculating machine invented by M. Thomas (of Colmar).*

Ever since men began to compute, endeavours have been made to facilitate calculations, or to perform them altogether, by means of mechanical contrivances. The history of these would afford to its author abundant and interesting materials. The hands and fingers offer the first and readiest help in computing. We can not only count on the fingers, but even perform complicated calculations. Take the following as an example: If we call the little finger of each hand 6, the next one 7 &c.; we may in a simple manner reduce the multiplication of two numbers between 6 and 10 inclusive, to the multiplication of two numbers under 5, (the multiplication by 10 being assumed). If we have to multiply two numbers 7 and 8 for example, we must hold the finger 7 of one hand against the finger 8 of the other, count the fingers which are held together and all those below them on both hands, and multiply the number by 10; then count for each hand separately the fingers above those which are held together, multiply these numbers together, and finally add the two products. In the above example, the number of the fingers which are held together and of those under them, is 5; and the number of the fingers above them, is on one hand 2, and on the other 3, from which it follows that

$$7\times 8=5\times 10+(2\times 3)=56.*$$

Coming now to Thomas's calculating machine,† we shall see that it can perform the first four rules of arithmetic with any numbers we choose,

* The proof of the general truth of this rule is very easily deduced from the formula:

$$ab=(a+b-10)10+(10-a)(10-b).$$

† For its construction see the ingenious treatise "Die Thomas'sche Rechen-maschine, von F. Reuleaux, Freiberg 1862."

whether integers or decimal fractions. From the fact that it is applicable to all numbers, the machine is of universal use; and possesses in consequence a great advantage, not only over Pascal's machine, which was based upon the special subdivisions of coin current in France in his time, but over the so-called Swedish calculating machine, which, having been invented by Babbage, was completed by the two Swedes, Scheutz and his son. The last being based on the principle of interpolation and the calculus of differences, only performs special calculations; but in these, its performances are almost incredible.

In practice the employment of Thomas's computing machine is principally of advantage, where we have to do with multiplications and divisions of large numbers, and where, without it, we should have to employ Tables of Logarithms; and in such cases the machine finishes the work in so short a time, that even a practised computer can scarcely find out the required logarithms within the same time. If it be a question of finding products, of which the sum is wanted, but the separate products are not required, then the machine performs the summation at the same time as the new multiplication,—nothing more being required for this purpose than to let the result of the previous computation remain. If we have computed for instance 3×4, the machine shows 12 as the result; if we let this number stand and compute 5×6, then the machine shows not 30, but $30 + 12 = 42$. Here we already save the addition of the separate products, but the employment of the machine is most advantageous when the products to be found and added together have all a common factor. This happens very frequently in computations connected with life insurance; at least, many formulæ admit of being suitably transformed for the purpose. Let us introduce, by way of example, the reserve for an ordinary life insurance. If we express this by $V_{x|n}$, where n denotes the number of years since the completion of the insurance, then we have

$$V_{x|1} = \frac{a_x - a_{x+1}}{1 + a_x} *$$

$$V_{x|2} = \frac{a_x - a_{x+1}}{1 + a_x} + \frac{a_{x+1} - a_{x+2}}{1 + a_x}$$

$$V_{x|3} = \frac{a_x - a_{x+1}}{1 + a_x} + \frac{a_{x+1} - a_{x+2}}{1 + a_x} + \frac{a_{x+2} - a_{x+3}}{1 + a_x},$$

or

$$V_{x|1} = \frac{a_x - a_{x+1}}{1 + a_x}$$

$$V_{x|2} = V_{x|1} + \frac{a_{x+1} - a_{x+2}}{1 + a_x}$$

$$V_{x|3} = V_{x|2} + \frac{a_{x+2} - a_{x+3}}{1 + a_x}$$

&c. &c.

* We have taken the liberty of transferring Dr. Zillmer's formulæ into the notation commonly used in this country. He writes the above formula thus

$$Res\ (1) = \frac{R(x) - R(x+1)}{R(x)}$$

R(x) being the annuity-due $= 1 + a_x$ or $= \ddot{a}_x$.—ED. *J. I. A.*

It follows from these formulæ that we have only to multiply the constant factor $\frac{1}{1+a_x}$ by the successive annuity-differences, without expunging the result of the previous multiplication, in order to obtain the values of the policy for different durations.

We have another example, where computation by logarithms is inconvenient, in the calculation of the premiums for an Endowment with returnable premiums. If the return of premiums is based upon the net premiums, we can put the formula for the premium in the form:

$$\frac{1}{n+(1-v)\cdot\frac{\mathbb{S}_x-\mathbb{S}_{x+n}-n\mathbb{N}_{x+n}}{\mathrm{D}_{x+n}}}\ *$$

where x is the age at entry, and n the number of years at the expiration of which the endowment is to be payable.

If we have to calculate by this formula the premiums for endowments payable at a given age for different ages at entry, we should calculate the second term of the denominator by itself. Here the factor

$$(1-v)\frac{1}{\mathrm{D}_{x+n}}$$

is constant for all ages at entry; and the difference between the variable factors for two successive ages at entry, as for example, x and $x+1$, is

$$\mathbb{S}_x-\mathbb{S}_{x+1}-\mathbb{N}_{x+n}$$

or

$$\mathbb{N}_x-\mathbb{N}_{x+n}.$$

If we first compute the values of this difference for all ages at entry, the calculation goes on very easily. If, for example, the endowment becomes payable at the age 24, we form in the first place the factor

$$\frac{1-v}{\mathrm{D}_{24}}$$

and multiply it successively and without expunging the results, first by D_{23}, then by $\mathrm{D}_{22}+\mathrm{D}_{23}$, then by $\mathrm{D}_{21}+\mathrm{D}_{22}+\mathrm{D}_{23}$, and so on.

To the separate results we add successively 1, 2, 3, &c.; and take the reciprocals of the sums. This last operation can be effected by the machine itself, namely by dividing 1 by the number in question. It is more convenient however to have a table of reciprocals, as for example the "Tables of the Reciprocals of Numbers from 1 to 100,000, by Lieut.-Col. W. H. Oakes. London, 1865."

Such a table is generally of great service, by enabling us immediately to replace each division by a multiplication; and with the machine, as with ordinary computations, multiplication is more convenient than division.

* Here $\mathbb{S}_x=\mathrm{S}_{x-1}$ (G. Davies).
$\mathbb{N}_x=\mathrm{N}_{x-1}$ (do.).

II.—11*th March*, 1868. DR. AUGUST WIEGAND delivered an address on the "*Antagonism between Theory and Practice*," of which the following is an abridgement:—

He stated that he did not intend to dwell upon the opposition of the over-confident practical man to theory; but rather to treat of the opposition which must of necessity sometimes arise between a perfectly true theory and a perfectly true practice. This, no doubt, might appear strange language for an actuary to employ; but he would go further, and say that in the business of life insurance, there are cases in which theory may bring an office to ruin, and only a prudent practice can save it. This prudent practice, which thus diverges from theory, may be best described by a phrase dating from March 1848, which is now a well-recognised rule in diplomacy. We must "take circumstances into account."

He then continued:—It is time I should give you an illustration. Follow me then to the sick-bed of an incurable consumptive patient: you will not be surprised if this consumptive man, who is insured in my office for a considerable sum, should say to me: "My dear sir, I should soon "get well, if I only had the means to enable me to spend next winter in a "warm climate: my physician is decidedly of that opinion, and I believe "him. If your Office would buy up my policy and not give me too small "a sum for it, I could then carry out my resolution. I insured six years "ago when 30 years of age for 4,000 Thalers—what can you afford to "give me?" Now, am I to answer this question as an actuary, or as a practical man? Shall I let my answer be dictated by strict theoretical rule, or shall I *take circumstances into account*? If I adhere to theory, I must answer him thus: "For the surrender of a policy, my Office allows "full three-fourths of the reserve, which, for your insurance, would amount "to 200 Thalers exactly"; and if he replied that he could not with 200 Thalers spend a winter in Nice, or Madeira, or even in Montreux, I might perhaps venture to go as far as the extreme theoretical limit, and offer him in addition the remaining one-fourth of the reserve. I should say: "That, "however, is the highest sum my Office can offer; for, out of the premiums "paid by you, we have nothing left but the policy-reserve: a part of the "premiums has gone in expenses of management, a second part in payment "of claims by death, and a third part, the smallest, has been carried to the "credit of the guarantee Fund, which every Office must have in order to "compensate for variations in the mortality of the lives insured." If I spoke strictly as an actuary, this must have been my answer; and it would have been quite right. That, however, which is right in theory, is, in this instance, a practical absurdity. The odds are a hundred to one that any one of you—without troubling himself the least about theory—would whisper to the sick man: "Don't accept those terms, I will give you 500 "Thalers with pleasure"—probably another among you would offer 600, and a third even 700 Thalers. Why? Because this is a case where you must take circumstances into account. And what are the circumstances in question? The man dies, in spite of Nice and Madeira; and before the year is out, the sum insured has to be paid: and 4,000 Thalers for 700, or even 800, if it must be so, is still a profitable transaction. These are the actual circumstances, and therefore the principle holds good "If you must choose between two evils, choose the lesser." Where the money is to

come from, whether it has been already provided or not out of the premiums, is no concern of mine. The policy must be bought, even if the owner will not let it go for less than 1,000 Thalers.

I now pass on to the subject which I have specially proposed to myself to investigate to-night, viz. : " the proper limits to a free disposing power " on the part of the insured over their property."

In the above example, theory may have seemed somewhat niggardly; but there are cases where theory would throw away money in a reckless manner, if not prevented by a prudent practice.

In theory, it is an axiom that the sum which is set aside out of the premiums as a reserve for any insurance, represents the credit of the policy-holder in the assets of the Company. This axiom is indisputable. If, however, any one should infer from it that the policyholder can under all circumstances freely dispose of his share of the assets, that would certainly be in accordance with theory, but by no means with a wise practice. In the case of the surrender of a policy, theory generally gives more than practice can sanction. Those who surrender their policies will evidently be mostly persons in sound health. But their withdrawal lowers the average state of health of those lives whose policies remain in force. For this reason, every Company must retain a part of the reserve as a compensation for the deterioration of the remaining risks. That there are exceptions in this respect, we have seen in the case of the man suffering from consumption; such cases however are very rare.

There are other branches of insurance business, in which Offices cannot allow any surrender value at all for a policy, or lend money upon it, without depriving themselves of the guarantees of their own stability. To this class belong all policies on one life, securing immediate or deferred annuities. The reason is clear. Such policies are only effected by a man who hopes to live a long time. Should such a change take place in his health, that his hopes of a long life are materially shaken, he cannot do better than surrender his policy. Let an Office once entertain proposals of this sort, and it would no longer be a question of its mortality agreeing with the law; for it would retain on its books only long-lived annuitants, whose annuities it could not pay out of the contributions previously received.

In the case of reversionary annuities instances may be imagined, in which the surrender of the policy is admissible; in the case, for instance, where the life insured should prove by medical evidence that his health is failing, and that the reversioner continues in the same state of health as when the policy was effected. Under such circumstances, however, proposals to surrender the policy will scarcely ever be made to the Office. For even if the husband whose health is impaired, wished to sell the policy, not having any presentiment of the true state of his health, his more prudent wife would certainly oppose it. Why then give a rule for such rare cases ?

Another case, where no surrender value can be given, is that of an Endowment without return of premiums. The reasons are just the same as those I have just given in regard to annuity policies. If the premiums are to be returned in the event of death, then the surrender is admissible; but the surrender value must be determined, not by the reserve, but solely by the bare amount of the premiums to be returned: in fact, just as if the policy were cancelled by the death of the life insured.

In all these cases, the assured possesses in theory the same full power of disposing of his policy-reserve as in all other kinds of insurance; but in practice, he cannot and must not have it. Is not however such a practice, a denial of the rights of the assured? Certainly not, for the assured retain the right of disposing of their interests, within certain limits, which are essential for the safety of the Office.

The only cases in which it is a matter of necessity that the assured should have the right of disposing of his policy-reserve are the two following: firstly, when the assured falls into difficulties and is unable to pay the premiums; and secondly, when, owing to unforeseen events, the object of the insurance has ceased. With regard to the first, every Office will most willingly re-purchase the policy, if the insurance is payable at death. If however it is an endowment, every Office will, with equal readiness, convert the existing insurance into one which requires no further payment of premiums, and secures to the assured notwithstanding, either a fixed sum or an annuity.

If the object of the insurance has ceased; for example, if the wife and children, who were to be provided for, have died before the husband, the Offices will readily exchange the insurance for a deferred annuity or an endowment.

From this you will perceive that the Offices are very far indeed from enriching themselves out of the premiums which the insured have perhaps saved with difficulty. But an absolutely free power of disposal over that which, according to theory, is the undeniable property of the assured, the Offices cannot guarantee without exposing themselves to a risk that would set all theory at defiance. Theory is still right, notwithstanding; but only for the whole collectively, never for a special case. In the same way as a man of the age of thirty has not the smallest guarantee that he will reach the age of 64, although the law of mortality allows him that expectation, every theoretical principle loses its meaning, if we seek to apply it to a particular case. Here we must not depend on calculations, but take the special circumstances into account.

There are in the world but few absolute truths: most of them are only relatively true. For that very reason therefore, theory and practice must not wish to be both in the right in each particular case, they must rather become reconciled. Theory is fatal without a judicious practice, but a practice which ignores theory, is simple destruction.

These remarks of Dr. Wiegand's appear to us eminently suggestive; but we cannot agree with him in thinking that a perfectly true theory can ever be in opposition to a perfectly true practice. In the instances he has quoted, it appears to us that practice is in advance of theory; and that the theory, far from being perfectly true, is decidedly imperfect. No doubt the theory is true for the whole of the lives insured collectively; and treating them all as *average lives*, makes a proper reserve for the whole. But when we come to particular cases, we know, as is very forcibly illustrated by Dr. Wiegand, that some of the lives insured are much *below the average* in their prospect of longevity; and that, on the other hand,

the majority of the lives insured are still such as for distinction we should call *select lives*—*i.e.*, such as would without difficulty be insured at the ordinary rate. We learn then, that since a larger reserve is certainly required for the *under-average* lives, a smaller reserve than the average is sufficient for the *above-average* or the *select* lives. But theory is not yet in a position to say what that smaller reserve should be. It is to be hoped that the valuable statistics as to the mortality among insured lives collected by the Council of the Institute of Actuaries, and shortly to be published, will enable this problem to be correctly solved, and throw much light upon many other important questions of a cognate character.—Ed. *J.I.A.*

Thirteenth Annual Report of the Insurance Commissioner of the Commonwealth of Massachusetts. January 1, 1868. *Part II. Life and Accident Insurance.*

IN the *Journal* for April last, we gave a *resumé* of the Eighth Annual Report of the Superintendent of the Insurance Department of New York, and in that for July, a detailed account of the Mutual Life Insurance Company of the same State. We believe that these contributions to our pages were not unacceptable to our readers, as serving to throw much new light on the condition of Assurance business on the other side of the Atlantic. It was seen that, in regard to Assurance as in everything else, our American cousins were not merely our rivals, but were fast becoming our teachers; that,—a new people, unencumbered with tradition and "the sacred dust of ages,"—their energetic and fruitful minds were striking out new ideas and methods to which we in this country, which had hitherto been regarded as the home if not the birth-place of Assurance, were entire strangers. These ideas and methods were not simply fantastic devices, the mere extravagancies of free thought, but were the product of sober reason and practical sense, and were directed, not only to the development of the Assurance principle, but also to the regulation of Assurance practice.

It is in view of this latter question—that of Assurance Government—that the able State paper which we now have under consideration, has for us at this moment a special interest. It has, indeed, another element of attractiveness, about which we shall probably say something. This is to be found in an elaborate essay by the learned Commissioner on the comparative merits of the "Per-